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Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find the Fourier series for the function $f(x) = x + x^2$ over the interval $-\pi \le x \le \pi$. Hence deduce that:
i) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots$ ii) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ **(07 Marks)**

b. Expand the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$ in half range Fourier cosine series. **(06 Marks)**

c. Find the constant term and the first two harmonics for the function $f(\theta)$ given by the following table: **(07 Marks)**

θ (in degrees)	0	60	120	180	240	300	360
$f(\theta)$	0.8	0.6	0.4	0.7	0.9	1.1	0.8

2 a. Show that the Fourier transform of the function
$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases} \quad \text{is} \quad F(\alpha) = \frac{2\sqrt{2}}{\alpha^3\sqrt{\pi}}(\sin\alpha - \alpha\cos\alpha).$$
Hence deduce that $\int_0^\infty \frac{\sin x - x\cos x}{x^3}\,dx = \frac{\pi}{4}$. **(07 Marks)**

b. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. **(06 Marks)**

c. If the Fourier sine transform of f(x) is given by $F_s(u) = \frac{\pi}{2}e^{-2u}$, find the function f(x). **(07 Marks)**

3 a. Find the various possible solutions of two-dimensional Laplace equation by method of separation of variables. **(07 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $u_{tt} = c^2u_{xx}$ subject to the conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(06 Marks)**

c. Solve the one-dimensional heat equation $c^2u_{xx} = u_t$, $0 < x < \pi$ subject to the conditions $u(0, t) = 0$, $u(\pi, t) = 0$, $u(x,0) = u_0 \sin x$ where u_0 is a non-zero constant. **(07 Marks)**

4 a. Find a curve of the best fit of the form $y = ax^b$ to the following data: **(07 Marks)**

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

b. For conducting a practical examination, the chemistry department of a college requires 10, 12 and 7 units of 3 chemicals x, y and z respectively. The chemicals are available in 2 types of boxes: Box A and Box B. Box A contains 3, 2 and 1 units of x, y, z respectively and cost Rs.300. Box B contains 1, 2 and 2 units of x, y, z respectively and costs Rs.200. Find how many boxes of each type should be bought by the department so that the total cost is minimum. Solve graphically. **(06 Marks)**

c. Solve the following LPP by simplex method:

Maximize $z = 2x_1 + 4x_2 + 3x_3$

Subject to the constraints $3x_1 + 4x_2 + 2x_3 \le 60$ $\quad 2x_1 + x_2 + 2x_3 \le 40$

$x_1 + 3x_2 + 2x_3 \le 80$ $\quad x_1, x_2, x_3 \ge 0$ **(07 Marks)**

PART – B

5 a. Use Newton-Raphson method to find an approximate root of the equation $x \log_{10} x = 1.2$ correct to 5 decimal places that is near 2.5. **(07 Marks)**

b. Use Relaxation method to solve the following system of linear equations:

$8x + 3y + 2z = 13$ $\quad x + 5y + z = 7$ $\quad 2x + y + 6z = 9$ **(06 Marks)**

c. Find the numerically largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ by power method taking $X^{(0)} = [1 \quad 0 \quad 0]^T$. Perform 6 iterations. **(07 Marks)**

6 a. Find the interpolating polynomial for the function y = f(x) given by f(0) = 1, f(1) = 2, f(2) = 1, f(3) = 10. Hence evaluate f(0.75) and f(2.5). **(07 Marks)**

b. Apply Lagrange's method to find the value of x corresponding to f(x) = 15 from the following data: **(06 Marks)**

x	5	6	9	11
f(x)	12	13	14	16

c. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}^{th}$ rule dividing the interval (0, 1) into 6 equal parts. Hence deduce the approximate value of π. **(07 Marks)**

7 a. Solve the wave equation $u_{tt} = 4u_{xx}$ subject to the conditions u(0, t) = 0, u(4, t) = 0, $u_t(x, 0) = 0$ and u(x, 0) = x(4 – x) by taking h = 1, k = 0.5 upto four steps. **(07 Marks)**

b. Find the numerical solution of the equation $u_{xx} = u_t$ when u(0, t) = 0, u(1, t) = 0, $t \ge 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$. Carryout computations for two levels taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$. **(07 Marks)**

c. Solve Laplace's equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in the following Fig.Q7(c).

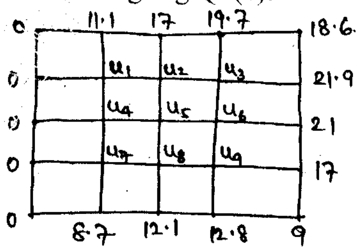

Fig.Q7(c) **(06 Marks)**

8 a. Find the z-transform of $5n^2 + 4\cos\frac{n\pi}{2} - 4^{n+2}$ and $\sinh n\theta$. **(06 Marks)**

b. Obtain in inverse z-transform of $\frac{z(2z+3)}{(z+2)(z-4)}$. **(07 Marks)**

c. Using z-transforms, solve $u_{n+2} + 3u_{n+1} + 2u_n = 3^n$ given $u_0 = 0$, $u_1 = 1$. **(07 Marks)**

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